IISER Mohali Swarnajayanti Fellow to fuse theories to understand symmetries of geometric objects

Dr. Mahender Singh, scientist from Indian Institute of Science Education and Research, Mohali is one of the recipients of the prestigious Swarnajayanti fellowship funded by the Department of Science & Technology.

One of the key aspects of mathematics is symmetry and the main aim of his proposal "Knots, Groups and Actions" for the Swarnajayanti Fellowship is to understand symmetries of interesting geometric and algebraic objects. The interaction of several areas of mathematics is a novelty of his proposal.

His team has been working broadly in topology and algebra, the two core areas of mathematics. In topology, they have been trying to understand algebraic aspects of knot invariants and structural aspects of compact group actions on manifolds. In algebra, they have been trying to understand automorphisms, conjugacy and cohomology of groups and other algebraic structures arising in low dimensional topology.

His study would span some of the central problems in algebraic topology, knot theory and group theory. These subjects weave together several areas of mathematics. Knot theory is a branch of topology that studies mathematical knots (defined as embeddings of a circle in the 3-dimensional Euclidean space) and interacts with physics and biology as well. While inspired by knots like those in shoelaces and rope, a mathematical knot differs in that the ends are joined together so that it cannot be undone.

Dr Singh plans to use ideas from combinatorial group theory and ring theory into knot theory, with an emphasis on finding newer properties of important knot invariants which helps distinguish knots. He will work on structural aspects of generalized braid groups and their planar analogues with a focus on finding newer invariants of their geometric realizations.

His recent works in topology have been on an important algebraic invariant of knots in the 3-space. He showed that this algebraic invariant satisfies some nice residual properties and has solvable word problem and also investigated its connectivity and transitivity properties.

In other recent works, Dr. Singh attempted to understand structure of finite groups that act freely on a class of manifolds. This direction stems from the classical problem of understanding the symmetries of spheres. He also proved several parametrised versions of the classical Borsuk-Ulam theorem.

Dr. Singh's recent work in algebra is a research monograph that unifies several ideas developed over the years around two classical problems in group theory concerning relations between orders of finite groups and that of their automorphism (a symmetry of the object, and a way of mapping the object to itself while preserving all of its structure) groups. He also investigated automorphisms and conjugacy classes in free groups, braid groups and certain Coxeter groups (an abstract group that admits a formal description in terms of reflections).

Dr. Singh along with his team will also train some M.S. students, Ph.D. students and Post-Doctoral Fellows under the fellowship.