MATHEMATICAL MODELS FOR SPREAD OF COVID-19: an explanation for non scientists

Joseph Samuel (RRI, ICTS) and Supurna Sinha (RRI)

Mathematical models are routinely used by Scientists to describe and predict natural phenomena. In the present covid-19 pandemic, we are all exposed to predictions from mathematical modelling of epidemics by experts. The predictions from these models sometimes differ widely, and it may be confusing to citizens and political leaders, who have to make important decisions based on these predictions. This article is written by two theoretical physicists, who feel that a popular exposition may be socially useful.

The main questions we address are :

1. What can we learn from these models?
2. How seriously should we take them?
3. Why do the predictions differ between models?
4. How are these models constructed?

WHAT CAN WE LEARN FROM THESE MODELS?

The most important idea from these models is that the initial growth of disease is ``exponential''. This word is used more often than it is understood. We will spend some time explaining what this means.

Let us suppose there is a bank that will give you 100% interest compounded daily. If you invest one rupee in such a bank. How much money would you have after a month (30 days)?

The answer is staggering: One hundred and Seven crores, thirty seven Lakhs, forty one thousand, eight hundred and twenty four!

This is the power of the geometrical progression, that all of us have studied in school. There is a charming folk tale (One Grain of Rice) of a girl who was granted a boon by the King. She asked for one grain of rice on the first square of a chessboard, two on the second, four on the third and so on till the 64th square. The king readily granted her boon, not realising that he had just promised five crore crore kilos of rice, far more rice than has ever been grown on the planet!

It is easy to underestimate the power of EXPONENTIAL GROWTH.
Infections due to
Covid-19 double every few days.
Numbers which appear small today may explode out of control in a few weeks.
When the US had only 15 cases, their president was complacent,

because15 seemed like a small number.
Six weeks
later, the number of cases exploded to far beyond the capacity of their medical resources and resulted in about 26000 deaths. A costly miscalculation!
The present number of infections in India seems small in comparison to global figures.
However, there is no room for complacency!

Exponential growth of infection in the early stages is a common prediction of all the mathematical models. The exponential growth stops only after a good fraction of the population is infected. The only way to slow the growth is to practise social distancing. This has the effect of increasing the time over which the infections double. Slowing the growth buys us time to increase our preparedness in terms of medical facilities, equipment, trained personnel, testing and contact tracing.

How seriously should we take the mathematical models?

All mathematical models are idealisations of the problem, which are based on simplifying assumptions. Simple models can capture qualitative features well and make predictions based on the value of a few parameters (for instance the doubling time), which can be gleaned from the past data. As the models get more complicated and ``realistic'', the number of parameters also increases. This results in a new kind of uncertainty stemming from our ignorance of a large number of parameters.

Small changes in the parameters can lead to large changes in the outcomes over a period of time. One should be suspicious of any prediction which gives precise dates and numbers: eg. if a lockdown period of 73 days is enforced, the epidemic will by controlled by June 21st and there will be a total of 170,641 deaths! Uncertainties in modelling preclude such certainties in prediction. However, skilful and honest use of models can provide rough estimates of outcomes resulting from a given course of action. One can have some confidence in qualitative predictions over a short time.

Why do the predictions differ between models?

As we said before, any model is based on a set of idealised assumptions. And to quote Professor Anthony Fauci, ``a model is only as good as the assumptions it is based on''. Different models make different assumptions and therefore make different predictions. In a social debate where different sections are advocating different strategies, it is possible that each faction will use models favourable to its cause.

How are these models constructed?

The simplest model is called the SIR model, which divides the population into three categories: Susceptible, Infected and Removed. When susceptible people interact with infected ones, there is a possibility of infection spreading which is given by a rate β.Some of the infected people recover and some others will die of the disease. In either case, they are regarded as removed from the study. This model ignores the possibility of reinfection and also asymptomatic infection.

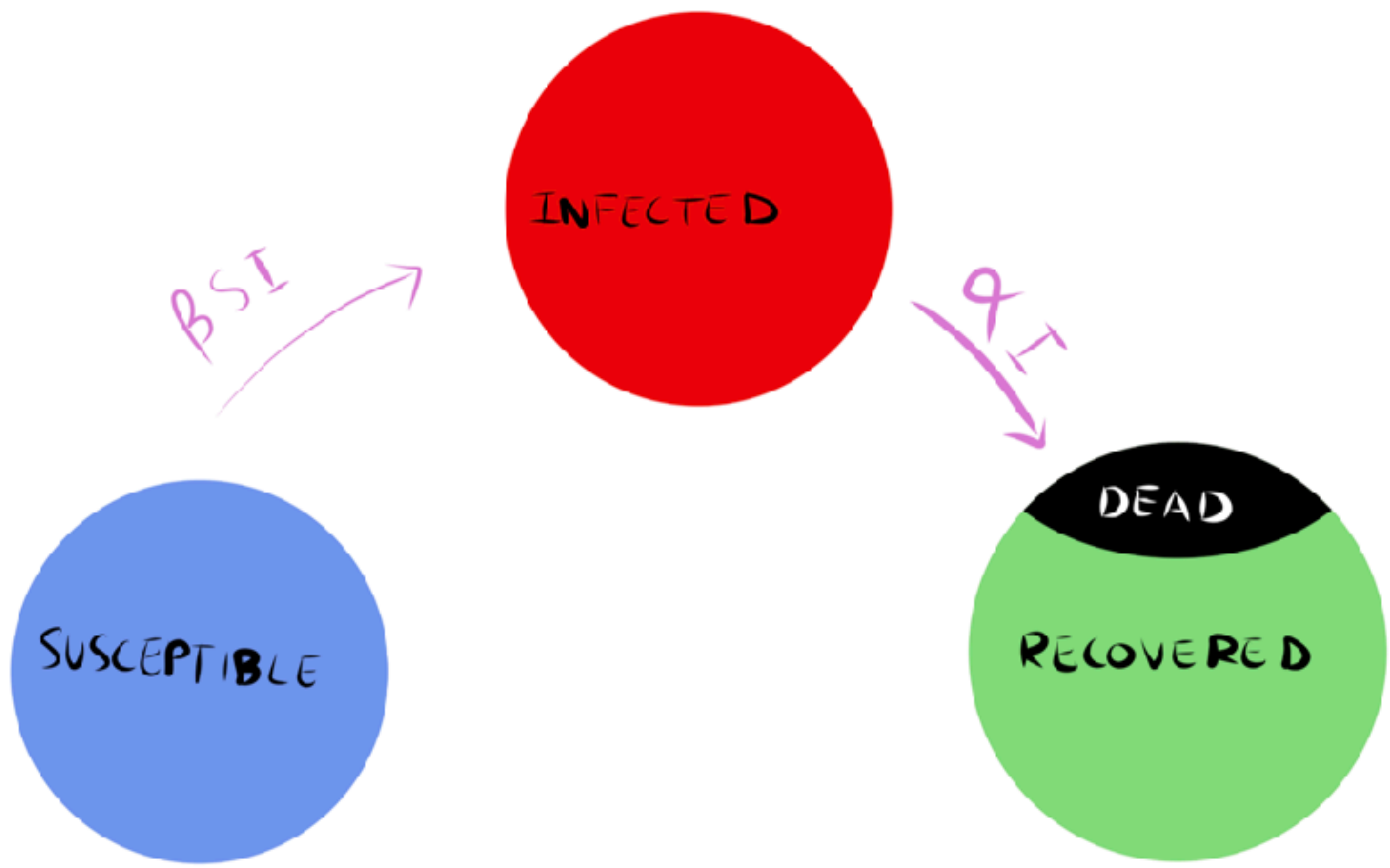

Figure shows the susceptible, infected and removed populations. The rates of infection and recovery/death are controlled by the parameters β and α. Social distancing lowers β.

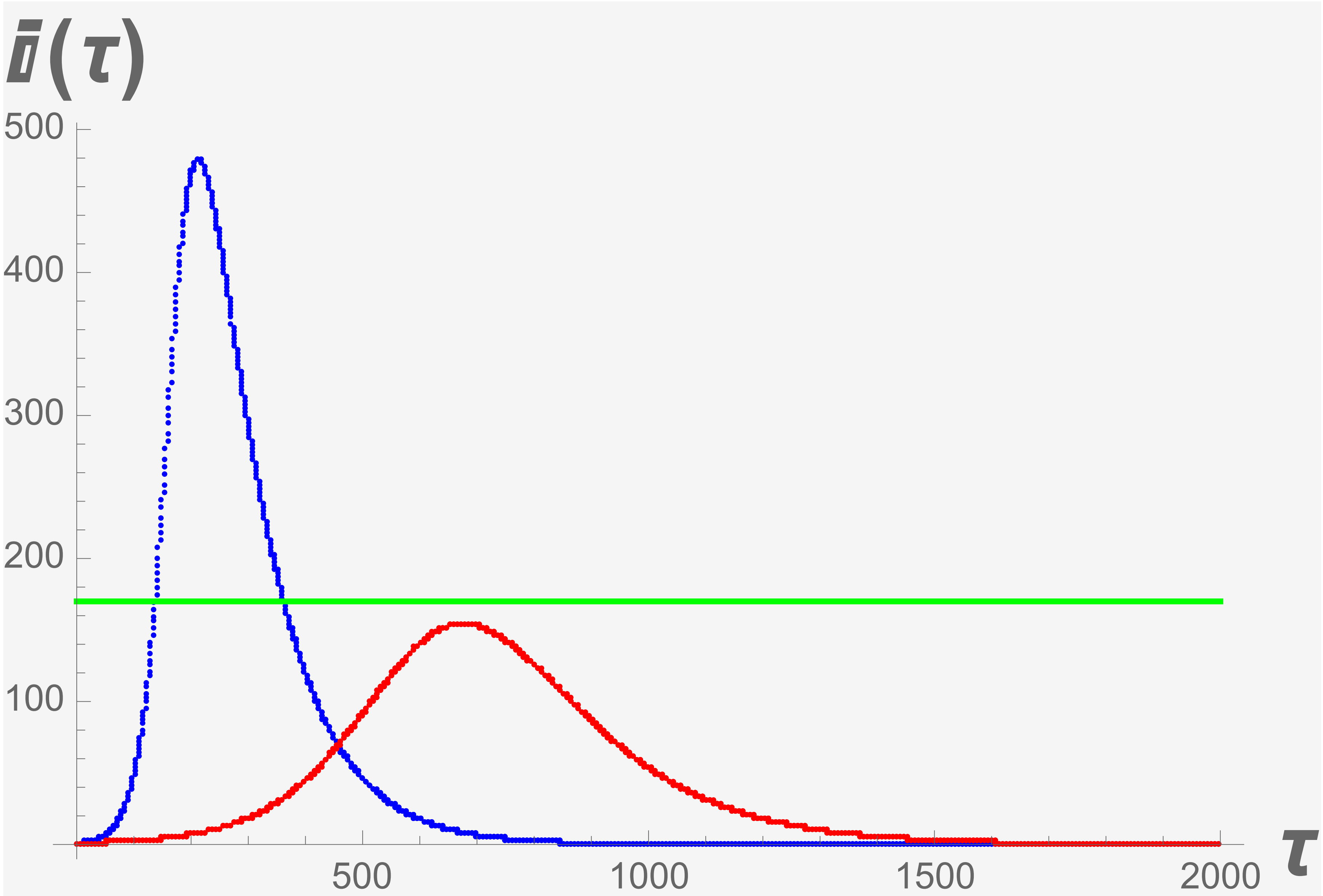

Figure (SIR model simulation) shows flattening of the curve of the number of infections with time. The initial blue curve gets flattened to the red curve if social distancing is implemented. The green line represents the hospital capacity.

To summarise, mathematical models are all based on simplifying assumptions. For instance, these models don't incorporate the effect of testing, contact tracing and isolation of infected population.
However, these mathematical models when used with care and good data, can help us to make rational decisions. They provide a rational, Science based approach to the social problem of infectious disease. Enlightened leadership based on consultation with epidemiologists can minimise the social cost in terms of death, suffering and economic hardship.

Since in India we are still at the beginning of the epidemic we can learn from the experience of other countries. Some countries like Taiwan and New Zealand offer positive examples, while the US is an example not to be followed. While it is instructive to learn from the examples of other countries, it is also important to keep in mind the problems specific to India, for instance, the issues of migrant labour, poverty, illiteracy and so on. In historical plagues, when the Science of disease and infection was unknown, people fell back on superstitions leading to a great toll of death and suffering. With the present knowledge of epidemiology, we as a society can fight the good fight rationally and win.

We thank Roshni Rebecca Samuel for help with the figures Abhishek Dhar and Suvrat Raju for a critical reading. April 16, 2020.